Cascading Failures in Smart Grid - Benefits of Distributed Generation

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***Abstract*—Smart grid is envisioned to incorporate local distributed power generation for better efficiency and flexibility. Distributed generation, when not used carefully, however, may compromise the stability of the grid. Recently, researchers have proposed innovative architectures (e.g., microgrid, LoCal grid) that virtualize a local generator as a constant load, source, or zero load to the grid, thus offering great promise to connect distributed generation into the grid without sacrificing its reliability. In fact, intuitively, using these architectures, distributed generation may enhance the stability of the power grid. In this paper, we develop a simulation model to quantify how much distributed generation can mitigate cascading failures. Applying this model to IEEE power grid test cases, we find that local power generation, even when only using a small number of local generators, can reduce the likelihood of cascading failures dramatically.**

I. Introduction

The electric grid is of vital importance to our daily life and work. With careful control and management, it has been operating with great reliability and robustness. However, large-scale cascading failures did occur, as evidenced by the most recent in 2003 when 50 million people in the U.S. Northeast and Southeastern Canada lost power for up to several days [1]. Maintaining power system reliability and reducing the risk of cascading blackouts is a critical issue.

Smart grid is envisioned to improve the efficiency, reliability, and flexibility of the current grid while reducing the rate at which additional electric utility infrastructure needs to be built [2]. To make this vision a reality, many advanced technologies need be incorporated into the current grid, leading to fundamental changes in the architecture of the grid. For instance, with distributed power generation (by local generators and renewable energy sources), the power grid is transformed from a centralized infrastructure with one-way power flow (from the centralized power plants to distributed loads) to a distributed infrastructure with two-way power flows.

Distributed generation, when not used carefully, may compromise the stability of the grid [3], [4]. Recently, innovative architectures (e.g., microgrid [5], LoCal grid [6]) have been proposed to virtualize a local generator as a constant load, source, or zero load to the grid, greatly simplifying its impact on the grid, and hence offering great promise to connect distributed generation into the grid without sacrificing the reliability of the grid. Indeed, when using these architectures, as loads are being served locally, less power flows inside the grid infrastructure. Therefore, intuitively, the stability and reliability of the grid can be enhanced. In this paper, we investigate whether this is indeed the case, and quantify how much improvement can be realized through distributed generation without any upgrade to the current grid infrastructure.

We focus our study on how distributed generation affects the likelihood of cascading failures, where the failures are triggered by an initial event (e.g., a transmission line failure), and then propagates inside the grid, leading to catastrophic large-scale blackouts. In particular, we develop a simulation model that investigates the vulnerability of the power grid to cascading failures as we gradually increase the number of local generators. Applying this model to IEEE power grid test cases, we find that local power generation, even when only introducing a small number of local generators into the grid, can reduce the likelihood of cascading failures dramatically. We also find diminishing gain from increasing the number of local generators: the benefits is dramatic at the beginning and less dramatic afterwards.

As related work, cascading failures in power grids have been analyzed and modeled in many studies (e.g., [7], [8], [9], [10], and see [11] for a review). Many methods have been proposed to mitigate and prevent cascading failures, e.g., using advanced decision techniques [12], [13], [14], [15], and combining monitoring and control tools [16], [17]. However, most existing studies consider traditional power grid with centralized generators and one-way power flow, while our study is in the context of smart grid, with the focus on how distributed generation mitigates cascading failures.

The rest of the paper is organized as follows. Section II presents the problem setting, and describes some background on power grid and cascading failures. Section III presents our methodology in investigating the benefits from local generation. Section IV describes simulation results over IEEE power grid test cases. Finally, Section V concludes the paper and presents future work.

II. Problem setting and background

In this section, we describe problem setting, and some background on power grids and cascading failure. The key notation is summarized in Table I for easy reference.

Consider an electric grid with n busses and m transmission lines. The busses are generators, load centers or substations. In addition to these n busses, there is a slack bus, at which the grid imports power from or exports power to a neighboring grid. With distributed generation, a bus can generate power to serve its load locally. Henceforth, we refer to the generators in the original grid as *central generators*, and a bus that generates power to serve its local load as a *local generator*. Adopting the architectures proposed in existing studies (e.g., [5], [6]), we assume a local generator virtualizes itself as a constant source, load, or zero load to the grid.

Let P_i denote the amount of power generated by bus i. When bus i is a central or local generator, $P_i > 0$; otherwise $P_i = 0$. Let L_i denote the load at bus i. It is positive when bus i has load and zero otherwise. Let $Q_i = P_i - L_i$. Then Q_i is the actual power injection from bus i to the grid: when $Q_i > 0$, bus i injects power into the grid, when $Q_i < 0$, bus i takes power out of the grid, and when $Q_i = 0$, bus i is self-sustained. The total generation and load of the grid needs to be balanced, i.e., $\sum_{i=1}^{n} Q_i + Q_s = 0$, where Q_s is the amount from the slack bus, and the sign of Q_s indicates whether the grid imports power from or exports power to a neighboring grid.

Let f_l denote the active power flow in line l. It is positive when flow is in the direction of "from" node to "to" node, and negative otherwise. The capacity of a line can be defined as the thermal limit, the voltage drop limit, or the steady-state stability limit [18], [19]. Of them, the steady-state stability limit is a practical estimate of line loadability and is relatively easy to estimate from the IEEE power grid test cases. Therefore, as in [9], we use the steady-state stability limit to represent the capacity of a line. More specifically, let f_l^c denote the capacity of line l. It is estimated as $0.475/x_l$, where $1/x_l$ is the susceptance of line l. For convenience, let $\mathbf{f}$ be a vector denoting the flows over all the lines. That is, $\mathbf{f} = [f_1, \ldots, f_m]^T$. Similarly, let $\mathbf{f}_c$ be a vector denoting the capacity for all the lines. That is $\mathbf{f}_c = [f_1^c, \ldots, f_m^c]^T$.

Individual links have circuit breakers to protect them from excessive local flows. For simplicity, we use a deterministic model for protection tripping. In particular, as in [9], we assume line protections for line l trip when $|f_l| > 1.25 f_l^c$, and refer to the value $1.25 f_l^c$ as the *protection trip point*.

For convenience, we define $f_l^n = f_l / f_l^c$ to represent the *loading* of line l. Let vector $\mathbf{f}_n = [f_1^n, \ldots, f_m^n]^T$ represent the loading for all the lines. Then

$$\mathbf{f}_n = \mathbf{f}/\mathbf{f}_c, \tag{1}$$

where the division is element-wise. We also use vector norms to represent the maximum loading and total loading (in absolute value) of the grid. In particular,

$$|\mathbf{f}_n|_\infty = \max(|f_1^n|, \ldots, |f_m^n|), \tag{2}$$

$$|\mathbf{f}_n|_1 = \sum_{i=1}^{m} |f_i^n|. \tag{3}$$

A. Estimating line power flows

We can use AC or DC power flow equations to estimate the line power flows. AC power flow equations are non-linear equations modeling the flows of both active and reactive powers, while DC power flow equations are linear equations modeling active power only. Assuming small voltage deviation (desirable for normal operation of the grid), DC power flow equations provide a good approximation of their AC counterparts. For simplicity, we use DC power flow equations in this paper.

TABLE I
KEY NOTATION.

Notation	Definition
n	Number of busses
m	Number of transmission lines
P_i	Amount of power generated by bus i
L_i	Amount of load at bus i
Q_i	Actual power injection from bus i to the grid
$\mathbf{f}$	Amount of active power flow in the transmission lines, $\mathbf{f} = [f_1, \ldots, f_m]^T$
$\mathbf{f}_c$	Capacity of transmission lines, $\mathbf{f}_c = [f_1^c, \ldots, f_m^c]^T$
$\mathbf{f}_n$	Loading of transmission lines, $\mathbf{f}_n = [f_1^n, \ldots, f_m^n]^T$

DC power flow equations can be found in many textbooks on power systems (e.g., [18], [19]). For completeness, we also describe them below. Let us first define some notation:

- Let $\mathbf{Q}$ be a vector denoting the actual power injection from busses. That is, $\mathbf{Q} = [Q_1, \ldots, Q_n]^T$.
- Let $\mathbf{A}_0$ denote the edge-node incidence matrix (i.e., rows are lines and columns are nodes), where an item $a_{ij} = 1$ if line i exits bus j, $a_{ij} = -1$ if line i enters bus j, and $a_{ij} = 0$ otherwise.
- Let $\mathbf{A}$ be a matrix reduced from $\mathbf{A}_0$. It is $\mathbf{A}_0$ with the column associated with the slack bus removed.
- Let $\mathbf{C}$ denote line property matrix. It is a diagonal matrix. More specifically, $\mathbf{C} = \text{diag}(1/x_i)$ where $1/x_i$ is the line susceptance of line i.
- Let $\mathbf{B}$ denote bus susceptance matrix, $\mathbf{B} = \mathbf{A}^T\mathbf{C}\mathbf{A}$.

Using the above notation, the DC power flow equations are

$$\mathbf{f} = \mathbf{C}\mathbf{A}\mathbf{B}^{-1}\mathbf{Q}. \tag{4}$$

B. Cascading failures

One transmission line outage may lead to a cascading sequence of line outages since when one line trips, its power flow redistributes to other lines in the grid, and may cause them to be congested and tripped. We obtain the evolution of cascading failures as follows [9]. Suppose a line, l, that connects bus i to j is tripped. Let $\mathbf{f}'$ be the vector of line flows after the outage of line l. Then

$$\mathbf{f}' = \mathbf{C}'\mathbf{A}\mathbf{B}'^{-1}\mathbf{Q}, \tag{5}$$

where $\mathbf{C}'$ is the line property matrix after the line outage, obtained by setting the l-th entry in $\mathbf{C}$ to be zero (i.e., assuming infinite impedance), and $\mathbf{B}'$ is the bus susceptance matrix after the line outage, $\mathbf{B}' = \mathbf{A}^T\mathbf{C}'\mathbf{A}$. After obtaining $\mathbf{f}'$, we check whether the flow in a line exceeds its protection trip point. If so, then this line trips, and we recalculate the power flow as described earlier. This procedure repeats until the grid breaks apart into disconnected islands, which generally lead to widespread blackout, or the outages self limit and stop before the grid breaks up.

We judge whether the grid breaks up or not by looking at the rank of the current bus susceptance matrix (i.e., $\mathbf{B}'$).

$$\text{maximize: } \sum_{i=1}^{n} P_i \quad (6)$$

subject to:

$$\mathbf{f} = \mathbf{C}\mathbf{A}\mathbf{B}^{-1}\mathbf{Q} \quad (7)$$

$$\mathbf{Q} = \mathbf{P} - \mathbf{L} \quad (8)$$

$$\mathbf{L} = y\mathbf{L}' \quad (9)$$

$$y > 0 \quad (10)$$

$$|\mathbf{f}| \leq \gamma \mathbf{f}_c \quad (11)$$

$$\sum_{i=1}^{n} P_i = \beta \sum_{i=1}^{n} L_i \quad (12)$$

$$P_i > 0, \text{ bus } i \text{ is a central generator} \quad (13)$$

$$P_i = 0, \text{ bus } i \text{ is not a generator} \quad (14)$$

Fig. 1. Determine the amount of load and generation at each bus in the original power grid (i.e., before adding local generators), where $\beta > 0, \gamma > 0, L_i' \in (0,1], \forall i$ are given beforehand.

$$\text{minimize: } |\mathbf{f}_n|_\infty \quad (15)$$

or

$$\text{minimize: } \frac{|\mathbf{f}_n|_1}{m} \quad (16)$$

subject to:

$$\mathbf{f}_n = \mathbf{f}/\mathbf{f}_c \quad (17)$$

$$\mathbf{f} = \mathbf{C}\mathbf{A}\mathbf{B}^{-1}\mathbf{Q} \quad (18)$$

$$\mathbf{Q} = \mathbf{P} - \mathbf{L} \quad (19)$$

$$|\mathbf{f}| \leq \gamma \mathbf{f}_c \quad (20)$$

$$\sum_{i=1}^{n} P_i = \beta \sum_{i=1}^{n} L_i \quad (21)$$

$$0 \leq P_i \leq P_i^0, \text{ bus } i \text{ is a central generator} \quad (22)$$

$$P_i = 0, \text{ bus } i \text{ is not a generator} \quad (23)$$

Fig. 2. Determine the amount of power generated by the central generators after adding local generators, where $\beta > 0, \gamma > 0$, $L_i, \forall i$, and the amount of power generated by each local generator are given beforehand.

More specifically, the grid breaks up when this matrix loses rank [20] (indeed, we need this matrix to be full rank and hence invertible to solve the power flow equations (4)).

III. Benefits from distributed generation

We develop a simulation model to investigate the benefits of using local generators in the power grid. This simulation model first determines the amount of load and generation at each bus in the original grid. It then adds k local generators to the grid, and these local generators and central generators jointly serve the loads in the grid. Afterwards, it simulates transmission line failure in both the original grid and the new grid with local generators to investigate their vulnerabilities to cascading failures. More specifically, each simulation run is as follows:

- In the original grid, for each bus i, we randomly generate a number, L_i', uniformly in $(0,1]$. We then solve the linear programming (LP) problem in Fig. 1 to determine the actual load, L_i, for each bus i, and the amount of power generated by each central generator.
- Suppose we add k local generators to the original grid. We randomly choose k load busses as local generators, and determine the amount of power generated by each local generator. After that, we solve the LP program in Fig. 2 to determine the amount of power generated by each central generator.
- In the original grid, we randomly pick a line and trip it out of service. We then follow the procedure in Section II-B to determine whether cascading failures caused by this line outage self limits or causes the grid to break up. We repeat the above process to simulate cascading failures in the new grid with local generators.

We make multiple simulation runs using independently generated seeds and use the percentage of runs ending up in grid breakup as the metric to measure the vulnerability of the grid with and without local generators.

We next describe the two LP problems in more detail. The LP problem in Fig. 1 determines the load and power injection at each bus in the original grid. In this problem, $\beta > 0$, $\gamma > 0$, and L_i' for each bus i are given beforehand. The variables to be determined are y (and hence the actual load $L_i = yL_i'$ at each bus i), and the amount of power generated by each central generator. In our simulations, we set γ close to 1.25 so that the most loaded line is close to its protection trip point since we are interested in cascading failures. The objective function maximizes the total amount of power that is generated by the central generators. Since power generation and demand have to be balanced, this optimization problem also maximizes the total amount of load that is supported by the grid. Constraints (7) to (10) specify how to calculate the flows inside the grid. Constraint (11) states that the amount of flow on each line is below γ times its capacity. Constraint (12) specifies the relationship between the total amount of power generation and load; the difference between these two quantities is compensated through the slack bus. Last, constraints (13) and (14) state that only central generators generate power.

The LP problem in Fig. 2 determines the amount of power generated by each central generator in the new grid (with local generators) so that the robustness of the new grid is maximized. In this problem, $\beta > 0$, $\gamma > 0$ are given beforehand, using the same values as those in the original grid. The load for each bus i, L_i, takes the same value as in the original grid. The amount of power generated by a

TABLE II
IEEE POWER SYSTEM TEST CASES.

Test case	busses	Lines	Load busses	Central generators
IEEE 14	14	20	9	4
IEEE 57	57	80	50	6
IEEE 118	118	186	64	53

local generator i, P_i, is pre-determined. In our simulation, we choose P_i uniformly randomly in $[1-\epsilon, 1+\epsilon]L_i, \epsilon > 0$ to match its load L_i since in many cases in practice, the primary goal of a local generator is to serve its load locally. The variables to be determined in the optimization problem are the amount of power generated by each global generator. The objective function is minimizing the maximum or average line loading (corresponding to objective function (15) and (16), respectively). The rationale for choosing these two objective functions is that when the line loading is low, the grid is not congested, and hence is more resilient to cascading failures. Constraints (17) to (21) specify how to calculate the flows inside the grid and the constraints on the flows. In Constraint (22), P_i^0 is the amount of power generated by a central generator i in the original grid (obtained by solving the LP problem in Fig. 1). This constraint specifies that the central generators in the grid are not upgraded to a higher capacity. Note that this optimization problem may not have feasible solutions. In our simulation, we found that only a very small fraction of the problem instances (less than 4%) have no feasible solutions. For those instances, slightly adjusting the value of β (by 1%) leads to feasible solutions.

In addition to the optimization problem in Fig. 2, we also investigate an *idealized* case where we are free to choose the amount of power generated by the local generators. The idealized case has less constraints than the problem in Fig. 2 (it does not have the constraint that the amount of power generated by each local generator is given beforehand), and hence provides a lower bound for each objective function in Fig. 2. This idealized case is guaranteed to have feasible solutions (since it has at lease one feasible solution where each central generator i generates an amount of P_i^0).

IV. SIMULATION RESULTS

We now investigate the benefits from using local generators in the power grid through simulation. Our simulation uses three IEEE power system test cases, IEEE 14 bus, IEEE 57 bus, and IEEE 118 bus [21]. These test cases all use a centralized one-way power flow architecture. Each test case contains one slack bus, several generator busses (central generators), and the rest are load busses. Properties of these test cases are summarized in Table II.

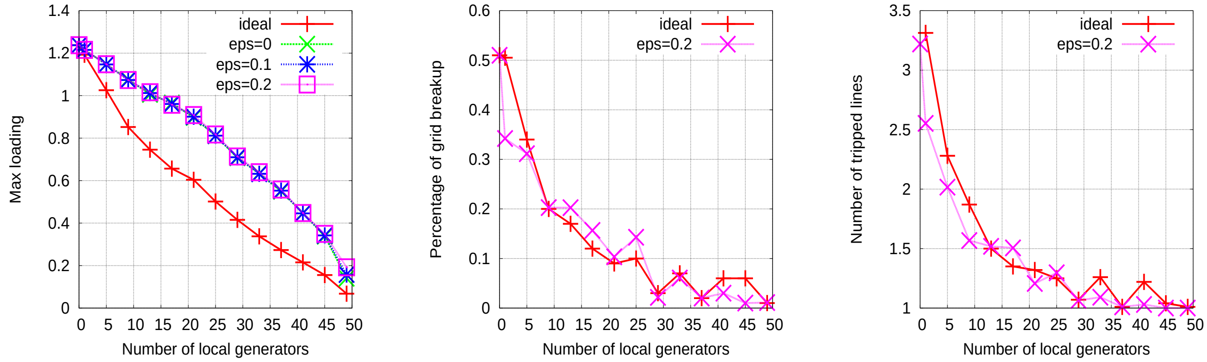
For each test case, we first look at the case with no local generators and solve the LP problem in Fig. 1. We then gradually increase the number of local generators, and solve the LP problem in Fig. 2. For each setting, we make 100 simulation runs. Each run differs in the amount of loads at the busses. The results we present are the average of these simulation runs. We set $\beta = 1.0$, i.e., the grid is self-sustained power network, and set $\gamma = 1.25 \times 0.99$ so that the most loaded line is close to its protection trip point, since we are interested in cascading failures. For local generators, we choose $\epsilon = 0, 0.1$ or 0.2. In addition, we investigate the idealized case where the amounts of power by local generators are not given beforehand. In the following, we only present the results under IEEE 57 bus and 118 bus test cases; results for 14 bus test case exhibit similar trend as IEEE 57 bus.

A. *Minimize maximum line loading*

Fig. 3 plots the results as we increase the number of local generators from 0 to 49 when the objective function is minimizing the maximum line loading (i.e., objective function (15)) for IEEE 57 bus test case. The results with zero local generator are for the original grid. Fig. 3(a) plots the maximum line loading for $\epsilon = 0, 0.1$ and 0.2, and the idealized case. The confidence intervals are tight and hence omitted. As expected, for each setting, the idealized case has a lower maximum line loading than the other cases. When $\epsilon = 0$, the amount of power generated by a local generator matches its load exactly; while a larger ϵ allows a larger deviation from the load. Results under the three different values of ϵ are very similar, indicating that ϵ has little impact on the result. For all the cases, we observe that the maximum line loading decreases monotonically as we increase the number of local generators. In particular, when most of the load busses are local generators, the maximum line loading is close to zero. This is expected, since in this case, most loads are served locally and hence the amount of flow inside the grid is very low.

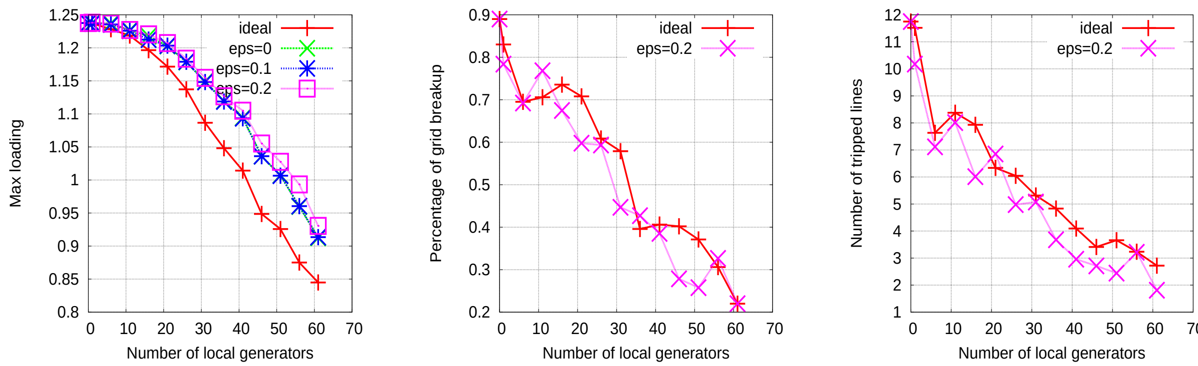
Fig. 3(b) plots the fraction of runs ending up in grid breakup versus the number of local generators. We only present results for $\epsilon = 0.2$ and the idealized case (the trends for $\epsilon = 0$ and 0.1 are similar). In general, we observe that the fraction of breakup instances decreases as we increase the number of local generators. In particular, even placing a small number of local generators dramatically decreases the likelihood of grid breakup. For instance, the fraction of grid breakup decreases from around 0.5 in the original grid to 0.3 when using five local generators. These results demonstrate that using local generators can significantly enhance the robustness of the grid to cascading failures. Fig. 3(c) plots the average number of tripped lines versus the number of local generators. We again observe the benefits of using local generators: the number of tripped lines decreases when increasing the number of local generators; and even a small number of local generators dramatically decreases the number of tripped lines.

Fig. 4 plots the results for IEEE 118 bus as we increase the number of local generators from 0 to 61. Fig. 4(a) shows a similar trend as Fig. 3(a): the maximum line loading decreases as we increase the number of local generators. However, unlike Fig. 3(a), the max line loading in Fig. 4(a) does not approach to zero. This is due to the large number of central generators in the IEEE 118 bus test case (nearly half of the busses are central generators) and Constraint (22): after solving the LP in Fig. 1, the amount of power generated by many central generators is zero, and Constrain (22) forces the generated



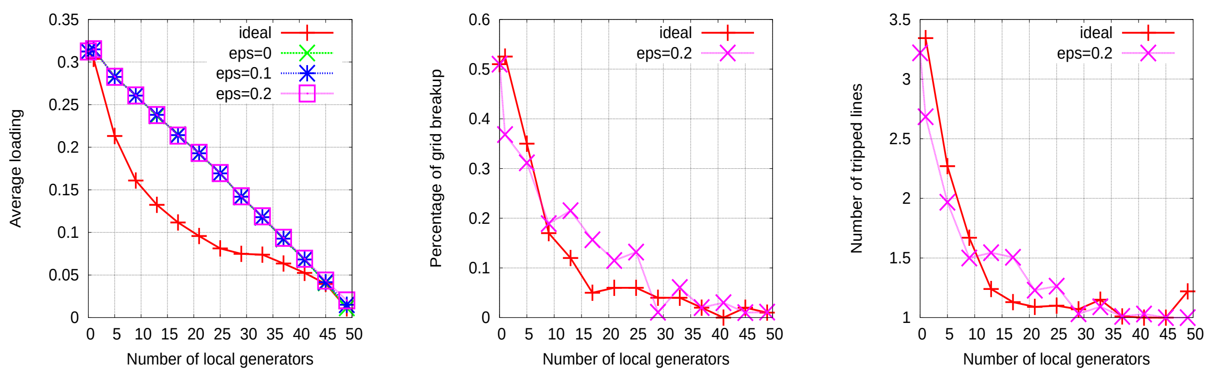
(a) Maximum line loading. (b) Percentage of grid breakup. (c) Number of tripped lines.

Fig. 3. Simulation results for minimizing the maximum line loading using the IEEE 57 bus test case.



(a) Maximum line loading. (b) Percentage of grid breakup. (c) Number of tripped lines.

Fig. 4. Simulation results for minimizing the maximum line loading using the IEEE 118 bus test case.



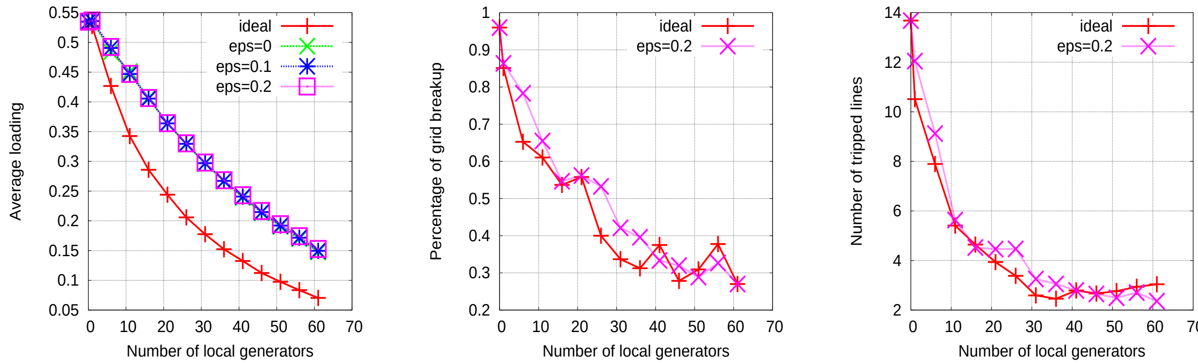
(a) Average line loading. (b) Percentage of grid breakup. (c) Number of tripped lines.

Fig. 5. Simulation results for minimizing the average line loading using the IEEE 57 bus test case.

amount at these central generators remain zero while solving the LP in Fig. 2. Therefore, even for a large number of local generators, the maximum line loading does not approach zero. For the same reason, although the percentage of grid breakup decreases sharply (see Fig. 4(b)) when increasing the number of local generators, it does not approach zero even with 61 local generators. Last, Fig. 4(c) shows a more dramatic decrease in the number of tripped lines than Fig. 3(c). This is due to the much larger number of transmission lines in the IEEE 118 bus case than the IEEE 57 bus case.

B. Minimize average line loading

Fig. 5 plots the results when the objective function is minimizing the average line loading (i.e., objective function (16)) for IEEE 57 bus test case. Again the results for zero local generator are for the original grid. In general, we observe similar trends as those in Fig. 3, indicating that the objective function of minimizing the average line loading is another effective way to enhance the robustness of the grid towards cascading failures. However, we also observe from Fig. 5 that a very small number of local generators may not lead to



(a) Average line loading. (b) Percentage of grid breakup. (c) Number of tripped lines.

Fig. 6. Simulation results for minimizing the average line loading using the IEEE 118 bus test case.

benefits. In particular, when there is a single local generator, the average line loading and the fraction of grid breakup are higher than those without local generators as shown in Figures 5(a) and (b), respectively. On the other hand, we again observe a diminishing gain from increasing the number of local generators in decreasing the percentage of grid breakup and the number of tripped lines. Fig. 6 plots the results for IEEE 118 bus, which shows similar trends as Fig. 5.

V. Conclusions and Future Work

In this paper, we developed a simulation model to quantify how much distributed generation can mitigate cascading failures as we gradually increase the number of local generators in the power grid. Applying this model to IEEE power grid test cases, we find that local power generation can reduce the likelihood of cascading failures dramatically. We also find diminishing gain from increasing the number of local generators: the benefits is dramatic at the beginning and less dramatic afterwards.

As future work, we will design other models to quantify the benefits of using local generation. For instance, how much it helps to increase the amount of load that can be supported by the power grid while still maintain the robustness of the grid. Furthermore, we will develop models that consider AC flows and voltage fluctuation (due to bidirectional power flows).

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