A Practical Joint Network-Channel Coding Scheme for Reliable Communication in Wireless Networks

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***Abstract*—In this paper, we propose a practical scheme, Non-Binary Joint Network-Channel Coding (NB-JNCC), for reliable multi-path multi-hop communication in arbitrary large-scale wireless networks. NB-JNCC seamlessly couples channel coding and network coding to effectively combat the detrimental effect of fading of wireless channels. Specifically, NB-JNCC combines non-binary irregular low-density parity-check (LDPC) channel coding and random linear network coding through iterative joint decoding, which helps to fully exploit the spatial diversity and redundancy residing in both channel codes and network codes. In addition, since it operates over a high order Galois field, NB-JNCC can be directly combined with high order modulation without the need of any bit-to-symbol conversion nor its inverse. Through both analysis and simulation, we demonstrate the significant performance improvement of NB-JNCC over other schemes.**

***Index Terms*—Network coding, non-binary channel coding.**

I. Introduction

COMPARED to wireline communication, wireless communication suffers from high and time-varying packet losses due to the detrimental effect of fading of wireless channels. One method to provide reliable communication is using redundant information to recover errors in the original information, which can be added either inside a packet (redundant bits/symbols at the physical layer) or across multiple packets (redundant packets at the network layer). The former is called *error correction* and the latter is called *erasure correction*. Specifically, channel coding is a conventional error-correction technique used for point-to-point communication over a single channel. It is implemented at the physical layer to recover erroneous bits/symbols through redundant parity-check bits/symbols appended to a packet. The error recovery capability depends on the specific coding strategy and the amount of redundant bits/symbols. On the other hand, erasure-correction is always used for end-to-end communication. It operates on the packet level, and can be used at either link layer, network layer, or application layer. Traditional network coding allows the intermediate nodes along multiple interleaved paths to generate redundant network-coded packets without decoding all original packets in a distributed manner [2], [3], although the redundancy on network layer can also be used for error correction.

Channel coding and network coding can be exploited simultaneously at the physical and network layers. Existing research on unifying channel coding and network coding can be roughly classified into the following three categories:

- *Separate network-channel coding*. For example, Larsson *et al.* [4] and Tran *et al.* [5] utilized network coding to implement Type-I Hybrid-ARQ for one-source multi-sink one-hop networks. Berger *et al.* theoretically analyzed the optimization problem in joint erasure-correction and error-correction coding schemes [6]. These studies treat two levels of codes separately and do not jointly exploit the redundant information.
- *Distributed channel coding*. Several approaches have been proposed to distribute the channel coding procedure to different nodes in a network. For example, Bao and Li proposed Adaptive Network Coded Cooperation (ANCC) for multiple transmitters sending data to a common receiver, where low-density generator matrix (LDGM) codes are dynamically constructed [7].
- *Joint network-channel coding*. Recently, many active studies jointly design network-channel codes to fully exploit the redundancy in both channel and network codes. For example, Bao and Li extended ANCC [7] to GANCC (Generalized Adaptive Network Coded Cooperation) on the packet level [8] and presented a general framework that unifies channel coding and network coding [9]. Hausl *et al.* presented iterative network and channel decoding on a Tanner graph [10], and also proposed joint network-channel coding for both multi-access relay channels and two-way relay channels where symbolwise combination of packets at the relay is replaced by judicious packet regeneration [11], [12]. Further, Duyck *et al.* proposed a structured full-diversity joint network-channel code in [13] and applied it in large networks [14]. Yang *et al.* and Kang *et al.* proposed iterative network and channel decoding when the relays cannot perfectly recover packets in [15] and [16], respectively. Zhang *et al.* applied LDPC codes and network coding to approach the capacity

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of two-way relay channel [17].

The aforementioned joint network-channel coding schemes were designed for small-scale wireless networks with specific topologies, which require the transmissions of each node to be well scheduled and the joint network-channel codes to be well designed.

In this paper, we focus on large-scale multi-path multi-hop wireless networks. In such networks, the transmission paths are determined by routing schemes dynamically and the intermediate relays process and forward packets in a distributed manner, thus relays cannot explicitly collaborate to construct a globally well-designed joint code. Specifically, we propose a *practical* joint network-channel coding scheme, *Non-Binary Joint Network-Channel Coding (NB-JNCC)*, which seamlessly couples non-binary irregular LDPC channel coding and non-binary random linear network coding. Different from existing joint network-channel code designs that focus on code optimization, the relays in this work employ the same nonbinary LDPC codes at the physical layer and generate the networked coded packets based on symbol-wise combination of incoming packets with randomly generated nonbinary coefficients. This allows the relays to process and forward packets independently without pre-scheduled collaboration, rendering the proposed solution suitable for large-scale multi-path multi-hop wireless networks.

The rest of the paper is organized as follows. In Sec. II we describe some preliminaries for the proposed NB-JNCC scheme. We then present the proposed scheme using a simple topology with two sources and two relays in Sec. III. After understanding the performance of the proposed scheme in this simple topology through both theoretical analysis and simulation study, we extend it to large-scale wireless networks in Sec. IV. We show that NB-JNCC achieves significant performance gains compared to other schemes. Sec. V concludes this paper and discusses future work.

II. Preliminaries

In this section, we first present the channel model, and then present background on the channel coding (in particular, low-density parity-check (LDPC) codes) and network coding that are used in our proposed NB-JNCC.

A. Channel Model

We assume that all lossy channels suffer from slow fading, i.e., fading keeps constant across one packet and varies from packet to packet independently (a.k.a. block fading). We model the channel as Rayleigh fading with additive white Gaussian noise (AWGN). More specifically, expressed in complex field $\mathbb{C}$,

$$y = hx + w, \tag{1}$$

where $y \in \mathbb{C}$, $x \in \mathbb{C}$ and $w \in \mathbb{C}$ denote the received signal, the transmitted signal and the additive noise, respectively, and $h \in \mathbb{C}$ denotes the fading coefficient. Since $|h|$ follows the Rayleigh distribution, $|h|^2$ follows an exponential distribution with the probability density function as

$$p(z) = \lambda e^{-\lambda z} \qquad (z = |h|^2). \tag{2}$$

The mean of $|h|^2$ is $1/\lambda$, i.e., $E\{|h|^2\} = 1/\lambda$, where $E\{\cdot\}$ denotes the expectation operation. Moreover, w is modeled as

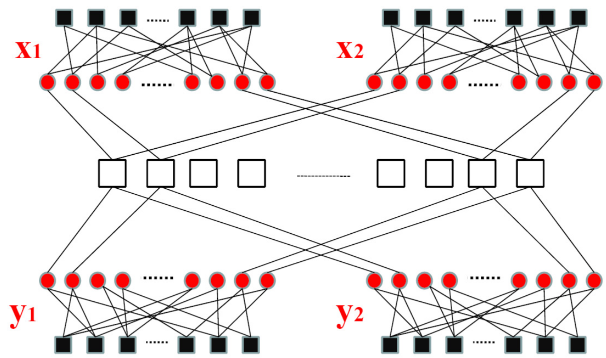


Fig. 1. Factor graph representation of the integrated code.

a zero-mean complex Gaussian random variable with variance N_0. Then the transmitted signal to noise ratio (SNR) can be defined as $\gamma = E_s/N_0$, where $E_s = E\{|x|^2\}$. Thus the instantaneous received signal to noise ratio is $\gamma|h|^2 = E_s|h|^2/N_0$ and the average received signal to noise ratio is $\gamma E\{|h|^2\} = E_s/\lambda N_0$.

B. Channel Coding

In NB-JNCC, we choose non-binary irregular low-density parity-check (LDPC) codes from [18] as the channel coding scheme. The reason for adopting this scheme is three-fold: 1) the LDPC codes can be graphically represented using factor graphs; 2) the adopted codes can approach the Shannon limit of various channels and can be encoded in linear time and in a parallel fashion; and 3) the channel coding/decoding on non-binary Galois field can be seamlessly combined with the network coding/decoding and underlying high order modulation.

An LDPC code [19], [20] is a linear error correcting code specified by a parity-check matrix $\mathbf{H}$ and a generator matrix $\mathbf{G}$, satisfying the relationship $\mathbf{G}\mathbf{H}^{\mathrm{T}} = \mathbf{0}$. Given $\mathbf{H}$, the corresponding generator matrix $\mathbf{G}$ can be obtained via Gaussian elimination. A source packet $\mathbf{u}$ of length k is encoded into a coded packet $\mathbf{x}$ through $\mathbf{x} = \mathbf{u}\mathbf{G}$, where $\mathbf{u}$ and $\mathbf{x}$ are row vectors. A key property of LDPC codes is that the parity-check matrix $\mathbf{H}$ is of low density in terms of the number of non-zero entries. An LDPC code can be represented using a sparse bipartite graph called Tanner graph as shown later in Fig. 1. Decoding of an LDPC code is done in an iterative manner via message passing along edges of its corresponding Tanner graph [21]. At the receiver side, the decoding process stops once the tentative copy $\hat{\mathbf{x}}$ satisfies $\mathbf{H}\hat{\mathbf{x}}^{\mathrm{T}} = \mathbf{0}$. The column weight distribution and row weight distribution of the $\mathbf{H}$ matrix highly affect the code's performance and complexity. Degree-distribution optimized LDPC codes can approach Shannon limit in various channels. In this paper, we use the nonbinary LDPC codes from [18], [22] whose parity check matrices consist of columns of weight 2 and columns of weight t ($t \geq 3$).

C. Network Coding

In NB-JNCC, we choose non-binary random linear network coding as the network coding scheme for the following reasons. Firstly, previous studies [2], [3], [23]–[26] have shown that random linear network coding is efficient and sufficient for error recovery. Secondly, the non-binary operation on a high order Galois field can provide independent network codes with high probability. Thirdly, the randomness of such network coding scheme renders itself applicable to large-scale networks as it allows distributed operation on each node without interrupting others.

In random linear network coding over a high order Galois field GF(2^q), the source generates the original packets, groups them into generations and linearly combines packets in a generation using randomly generated coefficients. More specifically, let $\mathbf{x}_1, \ldots, \mathbf{x}_K$ denote the K packets in a generation. The source linearly combines these K packets to compute K' ($K' \geq K$) outgoing packets, denoted as $\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_{K'}$ where $\mathbf{y}_i = \sum_{j=1}^{K} g_{ij}\mathbf{x}_j$. The coefficient g_{ij} is picked randomly from GF(2^q). The set of coefficients $(g_{i1}, \ldots, g_{iK})$ is referred as the *encoding vector* for $\mathbf{y}_i$ [3] and is carried in packets as overhead. A relay in the forwarding paths stores incoming packets from different routes in a local buffer for a certain period of time, then linearly combines the buffered packets belonging to the same generation. Suppose a relay, r, receives M incoming packets, $\mathbf{x}_1^r, \ldots, \mathbf{x}_M^r$. Let $(f_{i1}, \ldots, f_{iK})$ denote the encoding vector carried by $\mathbf{x}_i^r$, $i = 1, \ldots, M$. Since transmitting dependent packets is not useful for decoding at the sink, relay r encodes M' new packets, where M' is the rank of the coefficient matrix $[f_{ij}]$, $i = 1, \ldots, M$, $j = 1, \ldots, K$, and hence $M' \leq \min(M, K)$. Let $\mathbf{y}_1^r, \ldots, \mathbf{y}_{M'}^r$ denote the outgoing packets, $\mathbf{y}_i^r = \sum_{j=1}^{M} h_{ij}^r \mathbf{x}_j^r$, where h_{ij}^r is randomly selected from GF(2^q). Let $(g_{i1}^r, \ldots, g_{iK}^r)$ denote the encoding vector of $\mathbf{y}_i^r$, $i = 1, \ldots, M'$. Then $g_{ij}^r = \sum_{l=1}^{M} h_{il}^r f_{lj}$. The sink will receive multiple packets in the same generation. These packets are independent with high probability over high order Galois field, thus can be used to recover the original packets.

III. NB-JNCC Over a Two-source Two-relay Network

In this section, we present the coding and decoding procedures of NB-JNCC in a simple two-source two-relay topology. This topology allows us to thoroughly explain how NB-JNCC works and obtain analytical results on its performance. Operation of NB-JNCC in general large-scale wireless networks is deferred to Sec. IV.

A. Topology

The two-source two-relay topology is shown in Fig. 2. Two sources, S_1 and S_2, transmit two independent packets, $\mathbf{x}_1$ and $\mathbf{x}_2$, to a common sink, T. The transmissions of the sources are overheard by the relays (illustrated by the dashed lines). Upon overhearing $\mathbf{x}_1$ and $\mathbf{x}_2$, the two relays, R_1 and R_2, forward redundant packets, $\mathbf{y}_1$ and $\mathbf{y}_2$ to the sink, respectively. In this way, the sink will see four packets, $\mathbf{x}_1$ from source S_1, $\mathbf{x}_2$ from source S_2, $\mathbf{y}_1$ from relay R_1, and $\mathbf{y}_2$ from relay R_2. To focus on the joint decoding procedure at the sink, we assume the channels between the sources and the relays are lossless; our proposed scheme itself does not have such a constraint (see Sec. IV).

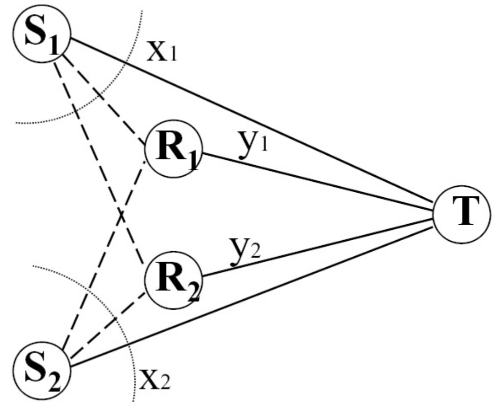


Fig. 2. A simple topology with two sources, two relays, and one sink.

The above topology differs fundamentally from the Multiple Access Relay Channel (MARC) topology [11] in that it uses two parallel relays instead of a single relay. All sources, relays and sink operate independently and cannot collaborate explicitly. This topology represents a basic component of transmissions over multiple interleaved paths in a large-scale wireless network, and helps to illustrate the distributed and collaborative operation among the relays, which is utilized by the proposed NB-JNCC scheme. Our objective here is not on the design of joint network-channel codes for the best performance subject to this particular topology; rather it is to elaborate the basic operation of relay nodes in a large-scale wireless network.

B. Code Construction

In traditional LDPC coding, the parity-check matrix $\mathbf{H}$ is designed first to guarantee the sparsity property, and the generator matrix $\mathbf{G}$ is derived accordingly. $\mathbf{H}$ should be agreed on by both the transmitter and the receiver or carried by the packet. As aforementioned, many studies [8], [11]–[14] used specific generator matrix at each relay to design joint network-channel codes with good equivalent parity-check matrix $\mathbf{H}$ for good performance and full diversity, where most of them required specific network topology and scheduling. While in a large-scale wireless network, there might be multiple interleaved paths from the source to the sink and a path may cross arbitrary hops and a relay may receive packets from arbitrary transmitters from different paths depending on the routing strategy. Thus, it is challenging to use individualized generator matrix at each node and optimize the joint network-channel codes throughout the whole network. For simplicity and scalability, we choose a common pair of $\mathbf{H}$ and $\mathbf{G}$ from a well-designed LDPC code [18] for all nodes, while network coding coefficients are generated at each node randomly and carried by the packet. The decoding complexity is guaranteed as described in Sec. III-C. The joint design for channel coding, network coding and routing is left in the future work.

We assume that source S_1 generates a packet $\mathbf{u}_1$ with k symbols (each of q bits) from Galois field GF(2^q), then encodes it into $\mathbf{x}_1$ using the common generator matrix $\mathbf{G}$ of size $k \times n$ as

$$\mathbf{x}_1 = \mathbf{u}_1\mathbf{G}, \quad (3)$$

where $\mathbf{x}_1$ and $\mathbf{u}_1$ are row vectors of length n and k respectively. Thus the channel code rate $r_c = k/n$. Similarly, the packet generated at source S_2 can be obtained as $\mathbf{x}_2 = \mathbf{u}_2\mathbf{G}$.

Assume packets $\mathbf{x}_1$ and $\mathbf{x}_2$ are broadcast respectively to the relays and the sink using orthogonal channels (at different time slots or via different frequencies). After receiving packets from the sources (recall that the channels between the sources and the relays are assumed to be lossless in this simple topology), the relays first decode and obtain the original packets, then generate packets using network coding and non-binary LDPC channel coding. The two network codes at relays R_1 and R_2 are represented as

$$\begin{aligned}\mathbf{y}_1 &= \alpha_{11}\mathbf{u}_1\mathbf{G} + \alpha_{12}\mathbf{u}_2\mathbf{G},\\ \mathbf{y}_2 &= \alpha_{21}\mathbf{u}_1\mathbf{G} + \alpha_{22}\mathbf{u}_2\mathbf{G},\end{aligned} \tag{4}$$

where the network coding coefficients α_{ij} $(i,j = 1,2)$ are drawn randomly from GF(2^q). Packets $\mathbf{y}_1$ and $\mathbf{y}_2$ will be sent to the sink from R_1 and R_2 respectively.

The sink receives four packets, $\mathbf{x}_1$, $\mathbf{x}_2$, $\mathbf{y}_1$ and $\mathbf{y}_2$, which forms a longer code

$$[\mathbf{x}_1\ \mathbf{x}_2\ \mathbf{y}_1\ \mathbf{y}_2] = [\mathbf{u}_1\ \mathbf{u}_2]\begin{bmatrix}\mathbf{G} & 0 & \alpha_{11}\mathbf{G} & \alpha_{21}\mathbf{G}\\ 0 & \mathbf{G} & \alpha_{12}\mathbf{G} & \alpha_{22}\mathbf{G}\end{bmatrix}. \tag{5}$$

Here we assume that the network coding coefficients can be conveyed to the sink without error. The code in (5) can be viewed as an integrated channel code with packets $[\mathbf{u}_1\ \mathbf{u}_2]$ and generator matrix $\mathbf{G}'$ as

$$\mathbf{G}' = \begin{bmatrix}\mathbf{G} & 0 & \alpha_{11}\mathbf{G} & \alpha_{21}\mathbf{G}\\ 0 & \mathbf{G} & \alpha_{12}\mathbf{G} & \alpha_{22}\mathbf{G}\end{bmatrix}. \tag{6}$$

We define the network code rate r_n as the fraction of original packets over all received packets at the sink. Then $r_n = 2/4$ for the scenario discussed above. Thus, the integrated code is of rate $r = r_c r_n = r_c/2$.

From (5), a straightforward method to retrieve the original packets is direct channel decoding. Given the equivalent generator matrix $\mathbf{G}'$, one can apply the Gaussian elimination algorithm (over an appropriate Galois field) to obtain the corresponding parity-check matrix $\mathbf{H}'$, which satisfies $\mathbf{H}'\mathbf{G}'^{\mathrm{T}} = \mathbf{0}$. One option for decoding is to adopt some version of belief propagation operating on $\mathbf{H}'$. However, it is usually hard and sometimes infeasible to perform this kind of decoding because the integrated belief propagation decoding is too complicated owing to the fact that $\mathbf{H}'$ is not sparse in general. Thus, we propose a simple iterative joint decoding algorithm, to be described next.

C. Iterative Joint Decoding

We propose a two-tier iterative joint network-channel decoding scheme, which implements soft decoding and allows information exchange inside and across packets.

1) Factor-Graph Representation: We rewrite (5) as

$$\begin{pmatrix}\mathbf{x}_1\\ \mathbf{x}_2\\ \mathbf{y}_1\\ \mathbf{y}_2\end{pmatrix} = \underbrace{\begin{pmatrix}1 & 0\\ 0 & 1\\ \alpha_{11} & \alpha_{12}\\ \alpha_{21} & \alpha_{22}\end{pmatrix}}_{\mathbf{M}}\underbrace{\begin{pmatrix}\mathbf{u}_1\\ \mathbf{u}_2\end{pmatrix}}_{\mathbf{U}}\mathbf{G}, \tag{7}$$

where $\mathbf{M}$ represents the coefficients used in network coding.

The integrated code can be represented by a factor graph as illustrated in Fig. 1. In this figure, the circles and dark filled rectangles represent symbol nodes and parity-check nodes of the channel coding, respectively, and the blank rectangles represent constraint nodes of the network coding. Each symbol node (circle) is connected to one network coding constraint node (blank rectangle) and several parity-check nodes (dark filled rectangles). In this way, all the symbols, either inside the same packet or across different packets, are connected directly or indirectly, allowing information to be exchanged among all symbols through the links (connections) and be jointly exploited for error recovery.

The proposed joint network-channel decoding relies on iterative message exchanges between two processing components, namely channel decoding and network decoding components. Specifically, as shown in Fig. 3, the *extrinsic* information from the network decoding component, L^e_{nc}, and the channel information, L_{ch}, are combined to generate the *a priori* information, L^a_{cc}, which is fed to the channel decoding; on the other hand, the *extrinsic* information from the channel decoding component, L^e_{cc}, and the channel information, L_{ch}, are combined to generate the *a priori* information, L^a_{nc}, which is fed to the network decoding component. The type of messages exchanged can be a probability mass function (pmf) over the Galois field or its log domain version (see [22] and the references therein).

The whole decoding process can start with either channel decoding or network decoding. When it starts with channel decoding, the extrinsic information from the network decoding component, L^e_{nc}, is empty and channel decoding is performed using only channel information, L_{ch}. Similarly, when it starts with network decoding, the extrinsic information from the channel decoding component, L^e_{ch}, is empty and network decoding is performed using only channel information, L_{ch}. In either case, we refer to an iteration with the two decoding processes (one after the other) as a *round*. The iteration continues until all the packets are decoded correctly or the maximum round is reached.

For brevity, we skip the description on the channel decoding component; see e.g., [21] for details on the sum-product algorithm. We next describe the network decoding component in more detail.

2) Network Decoding Component: The right part of Fig. 3 illustrates the network decoding for the k-th symbol, $x_{1,k}$ in packet $\mathbf{x}_1$, where for ease of exposition, $x^{nc}_{1,k}$ represents $x_{1,k}$ in the network decoding component. For network decoding, belief propagation based decoding algorithm is not applicable because in general the network coding matrix, $\mathbf{M}$, is not sparse. We therefore propose a *selection* updating rule for network decoding as described below. For the particular matrix $\mathbf{M}$ shown in (7), suppose that any two rows of $\mathbf{M}$ are linearly independent (this assumption holds for NB-JNCC with high probability over a high order Galois field), then any row can be represented as a linear combination of any two other rows. Thus, we can generate the *extrinsic* information for each packet using *a priori* information from two other packets. Here we choose the two packets that have smaller numbers of unsatisfied parity checks relative to other packets. Specifically, let us take the network updating of packet $\mathbf{x}_1$ as an example. Suppose that $\mathbf{x}_1$ can be represented as a linear combinations

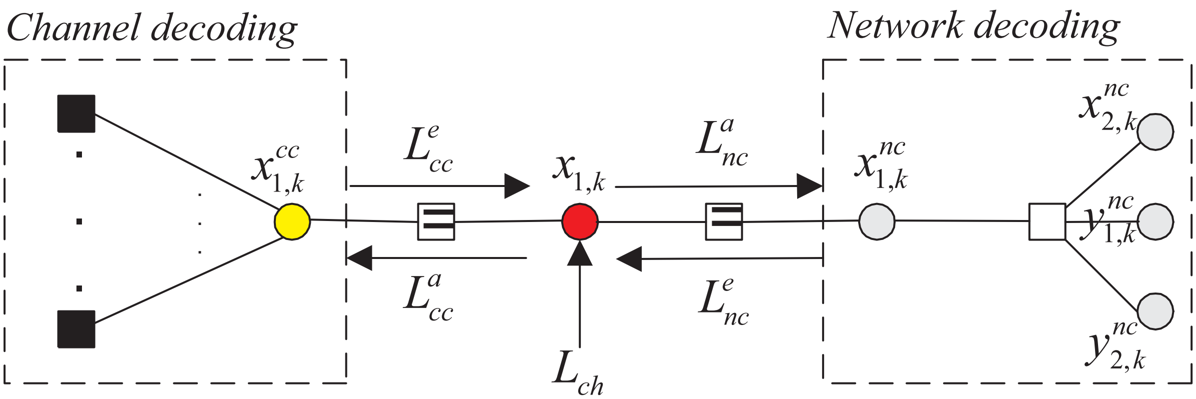


Fig. 3. Message exchange illustration for the k-th symbol $x_{1,k}$ of packet $\mathbf{x}_1$ between channel decoding and network decoding in the proposed iterative joint decoding of NB-JNCC. Circles represent symbol nodes, dark filled rectangles represent parity check constraints of channel coding, blank rectangles represent constraints of network coding, and a rectangle node with an equal sign inside means that the connected two symbol nodes are equal.

of two packets chosen from the three other packets, $\mathbf{x}_2$, $\mathbf{y}_1$ and $\mathbf{y}_2$. For instance $\mathbf{x}_2$ and $\mathbf{y}_1$ are better than $\mathbf{y}_2$ in the sense of having smaller numbers of unsatisfied parity checks, then we represent $\mathbf{x}_1$ as a linear combination of $\mathbf{x}_2$ and $\mathbf{y}_1$. The *a priori* information from $\mathbf{x}_2$ and $\mathbf{y}_1$ is used to generate the extrinsic information of $\mathbf{x}_1$, using the parity-check node updating rule in standard sum-product based LDPC decoding [22]. The above selection updating rule can be easily generalized to any network coding matrix $\mathbf{M}$.

D. Performance Evaluation

We evaluate the performance of NB-JNCC over the two-source two-relay topology (Fig. 2) using both analysis and simulation. We compare NB-JNCC with three other schemes

- *Binary Symbol-wise Joint Network-Channel Coding (BS-JNCC).* It differs from NB-JNCC only in that the network coding coefficients are chosen from GF(2). Note that better Binary JNCC schemes can be constructed by using linear transformations to form different channel codes on relays [8], [11]–[14], which we will not pursue in this paper as explained in Sec. III-B.
- *Direct Transmissions with Relays (DTR)*, where the sources directly transmit packets to the sink, and in addition relay R_i forwards the packets received from source S_i to the sink, $i = 1, 2$.
- *Direct Transmissions without Relays (DT)*, where the sources directly transmit packets to the sink, and the relays do not forward any packets. To make the comparison fair, in this scheme, we set the transmission power at the sources twice that in other schemes and the data rate to be the same.

1) Diversity Analysis: We treat the two packets, $\mathbf{u}_1$ and $\mathbf{u}_2$, as a *generation* of size two. Then the redundant packets in network coding allow the sink to recover the whole generation from a subset of received packets. This is due to the fact that packets transmitted through independent channels can provide *spatial diversity*. We next obtain packet error rate (PER), i.e., the probability that an original packet cannot be recovered, and generation error rate (GER), i.e., the probability when at least one packet in a generation cannot be recovered for all the schemes. For ease of analysis, we assume that all lossy links in Fig. 2 are identical and independent, with link outage probability P'_e under DT and P_e under other schemes, $P'_e < P_e$ since the sources in DT transmit at a power twice that in other schemes. Furthermore, we assume packet losses are independent. For DT, it is easy to see that

$$\mathrm{PER_{DT}} = P'_e, \tag{8}$$

and

$$\begin{aligned}\mathrm{GER_{DT}} &= 1 - (1 - \mathrm{PER_{DT}})^2 \\ &= 2P'_e - P'^2_e \\ &= 2P'_e + o(P'_e).\end{aligned} \tag{9}$$

For DTR, we have

$$\mathrm{PER_{DTR}} \approx P_e^2, \tag{10}$$

and

$$\begin{aligned}\mathrm{GER_{DTR}} &= 1 - (1 - \mathrm{PER_{DTR}})^2 \\ &\approx 2P_e^2 + o(P_e^2).\end{aligned} \tag{11}$$

Strictly speaking, (10) only provides an upper bound on PER in DTR, since the two identical packets from the source and the relay can be treated as repetition codes, and hence maximum-ratio combining can be performed before channel decoding.

For BS-JNCC, since the contents of the two network codes, $\mathbf{y}_1$ and $\mathbf{y}_2$, are identical, the network codes can be treated as one code, say $\mathbf{y}$. Then the outage probability $P_{\mathbf{y}} = \mathrm{PER_{DTR}} \approx P_e^2$. Without loss of generality, take packet $\mathbf{x}_1$ as an example. Packet $\mathbf{x}_1$ cannot be recovered only when itself and at least one of $\mathbf{x}_2$ and $\mathbf{y}$ are corrupted, yielding

$$\begin{aligned}\mathrm{PER_B} &\approx P_e(1 - (1 - P_{\mathbf{y}})(1 - P_e)) \\ &\approx P_e^2 + o(P_e^2).\end{aligned} \tag{12}$$

Similarly, a generation error will happen when more than two packets among $\mathbf{x}_1$, $\mathbf{x}_2$ and $\mathbf{y}$ are corrupted, yielding

$$\begin{aligned}\mathrm{GER_B} &\approx P_e^2 P_{\mathbf{y}} + 2P_e P_{\mathbf{y}}(1 - P_e) + P_e^2(1 - P_{\mathbf{y}}) \\ &\approx P_e^2 + o(P_e^2).\end{aligned} \tag{13}$$

For NB-JNCC, when the size of the Galois field is sufficiently large, any two packets can recover the whole generation. Take packet $\mathbf{x}_1$ as an example. It cannot be retrieved only when itself and more than two packets among $\mathbf{x}_2$, $\mathbf{y}_1$ and $\mathbf{y}_2$ are corrupted. Therefore

$$\mathrm{PER}_{\mathrm{NB}} \approx P_e(P_e^3 + \binom{3}{2} P_e^2(1-P_e)) = 3P_e^3 + o(P_e^3). \quad (14)$$

Since any two independent packets can recover the whole generation, the GER is the probability that at least three packets are corrupted. Therefore

$$\mathrm{GER}_{\mathrm{NB}} \approx P_e^4 + \binom{4}{3} P_e^3(1-P_e) = 4P_e^3 + o(P_e^3). \quad (15)$$

Discussion: Let *diversity order* be the dominating exponential power of the PER (or GER) expression. It describes the speed at which PER (or GER) changes when the channel SNR varies. We see from the above that DT has diversity order of 1, both DTR and BS-JNCC can reach diversity order of 2, while NB-JNCC achieves diversity order of 3, indicating that the error rates in NB-JNCC decrease much faster than those in other schemes. In fact, NB-JNCC achieves the optimal diversity order in the two-source two-relay topology. We should note that Binary JNCC, in contrast to BS-JNCC, can also achieve a diversity order of 3 if different channel codes on relays are formed through linear transformations.

2) Outage Probability Analysis: We consider two situations depending on whether relays are present or not: 1) direct network (corresponding to DT), where the sources directly transmit packets to the sink without any help from relays, and 2) relay network (corresponding to DTR, BS-JNCC and NB-JNCC), in which two relays assist the sources by transmitting redundant packets to the sink. We now investigate the maximum outage probability and the corresponding GER upper bounds that can be achieved using Shannon's coding theory.

As described in Sec. II, we assume all lossy links in Fig. 2 are independent and obey the same fading distribution with parameter λ. For the channel from sender i to receiver j, the instantaneous received signal to noise ratio is $\gamma|h_{i,j}|^2$, where $\gamma = E_s/N_0$ (see Sec. II-A), and $h_{i,j}$ is the fading coefficient of the channel. Let $I_{i,j}$ denote the maximum amount of information that can be carried on the channel. Then $I_{i,j} = \log_2(1+\gamma|h_{i,j}|^2)$.

In direct transmissions, according to Shannon's coding theory, we can directly obtain the necessary conditions for successful recovery of both packets in a generation as

$$\begin{cases} I_{\mathbf{x}_1} = \log_2(1+2\gamma|h_{S_1,T}|^2) > r_c \\ I_{\mathbf{x}_2} = \log_2(1+2\gamma|h_{S_2,T}|^2) > r_c \end{cases} \quad (16)$$

where 2γ accounts for the doubled transmission power at the sources, $h_{S_i,T}$ is the fading coefficient of the channel from S_i to T, $i = 1, 2$, and r_c is the channel code rate. Since two packets are independent, the corresponding GER bound is

$$\overline{\mathrm{GER}}_{\mathrm{direct}} = 1 - P(I_{\mathbf{x}_1} > r_c)P(I_{\mathbf{x}_2} > r_c). \quad (17)$$

In a relay network, with the assistance from relays, the outage probability of the network can be significantly increased. Since the network coded packets, $\mathbf{y}_1$ and $\mathbf{y}_2$, can carry partial information of the original packets $\mathbf{x}_1$ and $\mathbf{x}_2$, we obtain the following necessary conditions for successful recovery of a generation from the cut-set bound [27]

$$\begin{cases} \frac{1}{4}(I_{S_1,T} + I_{S_2,T} + I_{R_1,T} + I_{R_2,T}) > r \\ (I_{S_1,T} + \xi_{11} I_{R_1,T} + \xi_{21} I_{R_2,T}) > r_c \\ (I_{S_2,T} + \xi_{12} I_{R_1,T} + \xi_{22} I_{R_2,T}) > r_c \end{cases} \quad (18)$$

where $I_{S_i,T}$ and $I_{R_i,T}$ are the maximum amounts of information that can be carried on the channels from S_i to T, and from R_i to T, respectively, $i = 1, 2$, the coefficients $\xi_{11}, \xi_{12}, \xi_{21}, \xi_{22}$ are called *information splitters*, satisfying $\xi_{11} + \xi_{12} = 1$, $\xi_{21} + \xi_{22} = 1$, $\xi_{11}, \xi_{12}, \xi_{21}, \xi_{22} \in [0, 1]$, r and r_c represent the integrated code rate and channel code rate, respectively. The first inequality accounts for the necessary condition to recover the two original packets, and the other two represent the conditions to recover each packet with the help of the two relays, respectively.

In DTR, because $\mathbf{y}_1 = \mathbf{x}_1$ and $\mathbf{y}_2 = \mathbf{x}_2$, we have $\xi_{11} = \xi_{22} = 1$ and $\xi_{12} = \xi_{21} = 0$, and hence the last two inequalities imply the first one (recall $r = r_c/2$). In BS-JNCC and NB-JNCC, because the two original packets, $\mathbf{x}_1$ and $\mathbf{x}_2$, share the redundant information in the network codes, $\mathbf{y}_1$ and $\mathbf{y}_2$, the exact values of $\xi_{11}, \xi_{12}, \xi_{21}, \xi_{22}$ are difficult to decide and they may vary under different channel conditions. Thus, we obtain a loose upper bound of the system outage probability by weakening the conditions in (18) as

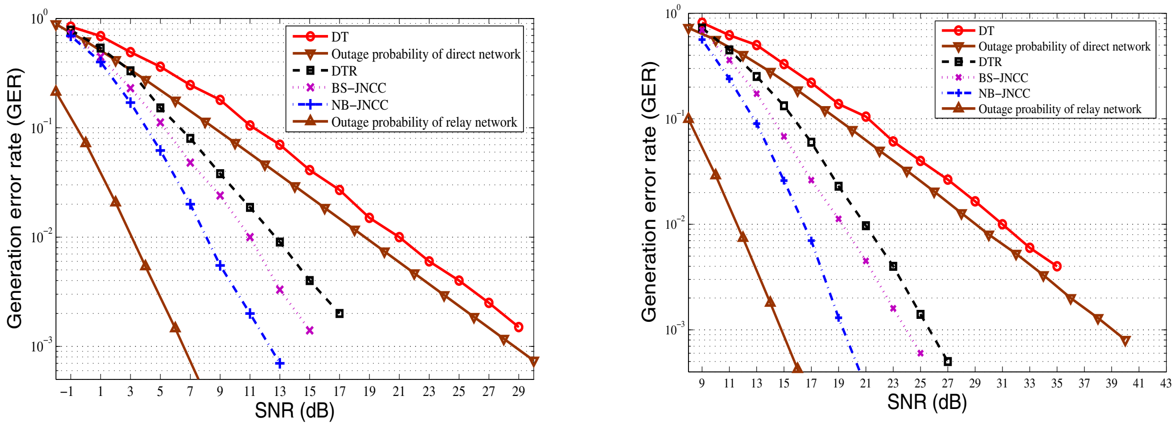
$$\begin{cases} I'_{(\mathbf{x}_1,\mathbf{x}_2)} = \frac{1}{4}(I_{S_1,T} + I_{S_2,T} + I_{R_1,T} + I_{R_2,T}) > r \\ I'_{\mathbf{x}_1} = (I_{S_1,T} + I_{R_1,T} + I_{R_2,T}) > r_c \\ I'_{\mathbf{x}_2} = (I_{S_2,T} + I_{R_1,T} + I_{R_2,T}) > r_c \end{cases} \quad (19)$$

and the corresponding upper bound for GER is

$$\overline{\mathrm{GER}}_{\mathrm{relay}} = 1 - P(I'_{(\mathbf{x}_1,\mathbf{x}_2)} > r)P(I'_{\mathbf{x}_1} > r_c)P(I'_{\mathbf{x}_2} > r_c). \quad (20)$$

3) Simulation results: We now present extensive simulation results to demonstrate the benefits of NB-JNCC. In our simulation, we set $k = 800$, $r_c = 0.8$, $q = 4$, i.e., the original packets from the sources, $\mathbf{u}_1$ and $\mathbf{u}_2$, contain 800 symbols, each of 4 bits, and the encoded packets using non-binary irregular LDPC contain $n = 1000$ symbols. The average column weight of the non-binary irregular LDPC code is 2.8 [18]. In NB-JNCC, the coefficients in network coding can be randomly drawn from GF(2^4) [25]. For simplicity, we fix the coefficients to $\alpha_{11} = \alpha_{12} = 7$, $\alpha_{21} = 12$, and $\alpha_{22} = 13$. All lossy links in the two-source two-relay topology are independent and have the same fading distribution. We obtain results using both BPSK and 16QAM modulation. The channel decoding component runs up to $L = 6$ iterations in each round. Our preliminary version [1] suggests to determine L dynamically according to the actual channel conditions.

Overall Performance Comparison. Fig. 4(a) and (b) compare the GER of the various schemes when varying the average received signal to noise ratio (SNR) under BPSK and 16QAM modulations. In Fig. 4(a), we observe that NB-JNCC outperforms other schemes under all metrics, especially under high SNR. Specifically, at GER of 10^{-3}, NB-JNCC outperforms DT by about 20 dB, and outperforms the other two schemes by 3 to 5 dB. In addition, Fig. 4(a) demonstrates that the diversity orders of the various schemes from the simulation agree very



(a) GER under BPSK. (b) GER under 16QAM.

Fig. 4. Generation error rate of NB-JNCC and other schemes under BPSK and 16QAM modulations.

well with our analytical results: DT has a diversity order of 1, DTR and BS-JNCC have a diversity order of 2, and NB-JNCC achieves a diversity order of 3. We also note that although BS-JNCC uses network coding to exploit the cooperation between packets, it only outperforms DTR in terms of GER, while shows no performance gains in terms of PER (figure on PER omitted). This observation can be verified through (10)-(13).

In Fig. 4(a), we also plot the bounds of outage probability obtained from (17) and (20) through numerical integration. The outage probability limit of direct networks indicates the best performance achievable by using channel coding alone. We observe DT approaches this bound within 3 dB. The outage probability limit of relay networks represents an upper bound for DTR, BS-JNCC and NB-JNCC. We see that NB-JNCC is the closest to this upper bound with about 6 dB performance loss. Furthermore, compared to other schemes, NB-JNCC shares the same diversity order with the upper bound, indicating that it fully exploits the diversity of the network.

Practical communication systems often employ high order modulation to increase the spectral efficiency. Using modulations having the same size as the Galois field, each coded symbol can be directly mapped onto a constellation point. Hence, all coding/decoding and modulation/demodulation processes can be unified without the need of bit-to-symbol conversion and its inverse. Fig. 4(b) presents the results with 16QAM modulation, which leads to a spectral efficiency of $4r = 1.6$ bits/s/Hz compared to $r = 0.4$ bits/s/Hz for BPSK modulation. Accounting for the fact that each bit transmission only gets $1/4$ of the symbol power, the results under 16QAM are comparable with those under BPSK, and all conclusions for BPSK hold true for 16QAM modulation. The gap between NB-JNCC and the upper bound of outage probability with 16QAM is about 4 dB, showing a gain about 2 dB over the case with BPSK, because 16QAM is more bandwidth efficient than BPSK.

Joint Decoding Gain. Non-binary separate network-channel coding (NB-SNCC) schemes treat channel codes and network codes separately, where channel decoding is followed by network decoding with no iteration, while NB-JNCC fully exploits the redundancy both inside and across packets through iterative joint decoding. For NB-SNCC, the soft information in packets that fail channel decoding is not exported to other packets, and hence is wasted. Fig. 5 demonstrates the advantage of NB-JNCC over NB-SNCC. Although both schemes have the same diversity order, NB-JNCC outperforms NB-SNCC by about 2 dB on average.

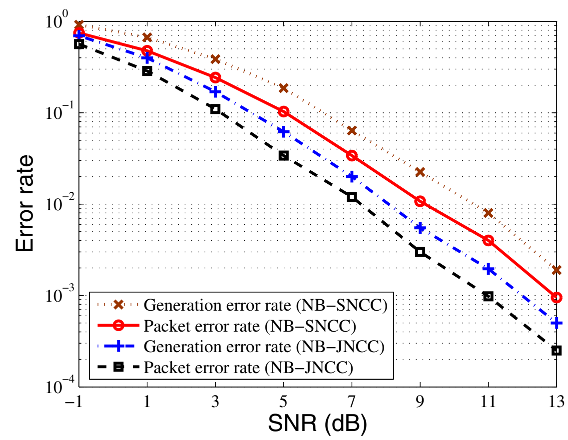


Fig. 5. Performance gain of joint decoding over separate decoding.

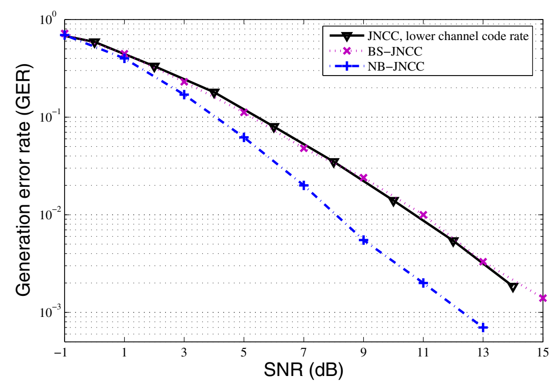


Fig. 6. Performance of different schemes under the same overall code rate.

Where to Put the Redundancy? NB-JNCC exploits the redundancy in both channel codes and network codes. An interesting question is: with a fixed total code rate $r = r_c r_n$, how shall we split redundancy between the channel codes and network codes? All the simulations above use $r_c = 0.8$ and $r_n = 0.5$. We now explore another way to split the redundancy. Specifically, by removing one relay, we can reduce the network code rate r_n to $2/3$, and put more redundancy in channel coding with channel code rate $r_c = 0.6$, so that the total code rate r is unchanged. The performance comparison is shown in Fig. 6. Having only one relay, the new scheme can achieve a diversity order of at most 2. We observe that the performance of the new scheme is no better than that of BS-JNCC with two relays, and much worse than that of NB-JNCC with two relays. This observation is consistent with the conclusion in [6] (which shows that to optimize the code rate assignment in joint error- and erasure-correction coding, more redundancy across packets is desirable in severe fading channels). Since typically multiple relays are available in large-scale networks, it is beneficial to exploit network cooperations rather than putting too much effort on protecting a single channel.

IV. NB-JNCC Over Large-scale Wireless Networks

We have demonstrated the benefits of NB-JNCC through both analysis and simulation in the two-source two-relay network. In this section, we extend NB-JNCC to large-scale networks, and evaluate its performance through extensive simulation.

A. Coding and Decoding Procedures

The operations of NB-JNCC in large-scale wireless networks involve three types of nodes: source, intermediate relays and sink. The source is responsible for generating packets and grouping them into generations. A source and an intermediate relay both perform channel coding and network coding to generate packets to be forwarded. After receiving packets from different paths, an intermediate relay can recover the original packets using joint network channel decoding when possible and encode new packets. A sink performs joint network channel decoding to retrieve the original packets.

Without loss of generality, assume that a source generates K packets represented as $\mathbf{u}_i$ $(i = 1, \ldots, K)$; each packet contains k symbols from GF(2^q). These K packets will be first encoded using non-binary irregular LDPC coding, then encoded by random linear network coding before being injected into the network. For simplicity, we assume that all nodes use the same LDPC generator matrix $\mathbf{G}$.

For an intermediate relay, suppose that it receives M packets, $\mathbf{x}_1, \ldots, \mathbf{x}_M$, which are already network encoded and can be expressed as $\mathbf{x}_i = \sum_{j=1}^{K} \alpha_{ij} \mathbf{u}_j \mathbf{G}$. Thus, we have

$$\begin{pmatrix} \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_M \end{pmatrix} = \begin{pmatrix} \alpha_{11} & \ldots & \alpha_{1K} \\ \vdots & \vdots & \vdots \\ \alpha_{M1} & \ldots & \alpha_{MK} \end{pmatrix} \begin{pmatrix} \mathbf{u}_1 \\ \vdots \\ \mathbf{u}_K \end{pmatrix} \mathbf{G}. \quad (21)$$

The network coding matrix $\mathbf{M} = [\alpha_{ij}]$ is of size $M \times K$. Assume that $\mathbf{M}$ has a rank of K', $K' \leq \min(K, M)$, which means that K' packets are independent. The intermediate node can recover at most K' independent packets. If $K' = M$, all packets are independent and none of them can be represented by others, then channel decoding is applied to each packet separately. If $K' < M$, there exist redundant packets and the proposed iterative joint network-channel decoding can be applied to exploit the redundancy for error recovery. Assume that M' packets are successfully decoded as $\mathbf{x}'_i = \sum_{j=1}^{K} \alpha'_{ij} \mathbf{u}_j \mathbf{G}$. Then the relay can regenerate arbitrary N (depending on the specific schemes) packets using random linear network coding as $\mathbf{y}_i = \sum_{j=1}^{M'} \beta_{ij} \mathbf{x}'_j = \sum_{k=1}^{K} \gamma_{ik} \mathbf{u}_k \mathbf{G}$, $1 \leq i \leq N$, where β_{ij} is randomly drawn from GF(2^q) and $\gamma_{ik} = \sum_{j=1}^{M'} \beta_{ij} \alpha'_{jk}$. In this way, nodes in the network can work in a distributed manner without explicit cooperation. At each node, newly received packets can be directly added to the decoding procedure without interrupting other received packets.

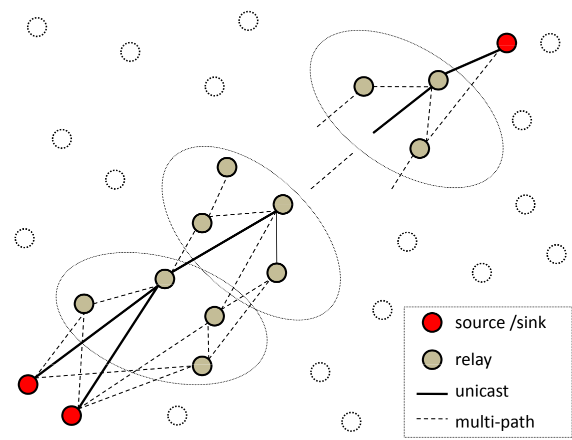


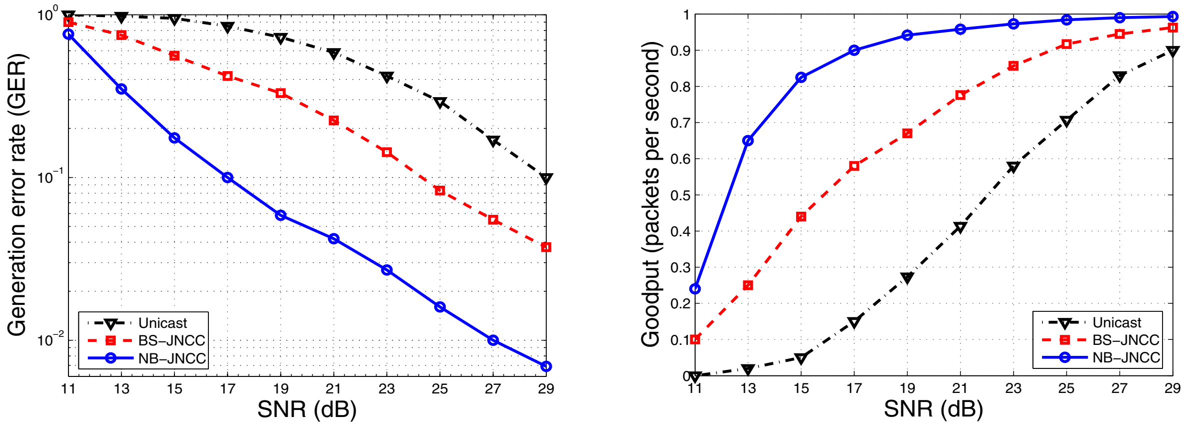
Fig. 7. Multi-path routing in a large-scale wireless network.

At the sink, to recover a generation of the original packets, the number of received packets should be no less than K and the corresponding network coding matrix should have full-column rank. Therefore, an appropriate routing algorithm should be used to guarantee that the sink receives a sufficient number of packets. Otherwise, retransmission should be triggered.

B. Packet Transmission in Large-scale Wireless Networks

The efficiency of NB-JNCC relies on the quality of the underlying paths determined by a multi-path routing algorithm. In NB-JNCC, intermediate nodes encode incoming packets into new packets before forwarding them. Ideally, each intermediate node should obtain as many diverse packets as possible to provide sufficient number of independent packets. To fully utilize this property and avoid transmitting too many packets, the underlying routing scheme should render multiple interleaved paths.

NB-JNCC can be used on top of any routing protocol that provides multiple interleaved paths. Without limiting ourselves to a particular multipath routing protocol, we model multipath routing in a large-scale wireless network through *relay sets* [23] as illustrated in Fig. 7. The source (or multiple collaborating sources) broadcasts the packets to its downstream neighbors (i.e., nodes within its transmission range and in the forwarding paths), referred to as a *relay set*. Nodes in the relay set further forward the packets to their neighbors,



(a) Generation error rate. (b) Goodput.

Fig. 8. Performance comparison in wireless networks with different channel conditions. There are three relay sets from the source to the sink and four relays in each set.

forming another relay set. Guo *et. al.* showed that the relay sets should have similar number of nodes to obtain the benefits of network coding and propose *path or redundancy adaptation* so that the relay sets are of similar sizes [23]. In our simulation, we assume the relay sets contain the same number of relays.

C. Performance Evaluation

Our performance evaluation of NB-JNCC in large-scale networks is through simulation (analytical study is very challenging, and is left as future work). On the other hand, as we shall see, our simulation results in large-scale networks are consistent with the analytical and simulation results in the simple two-source two-relay network.

We compare the performance of NB-JNCC and other schemes using the network setting illustrated in Fig. 7. In this setting, two sources collaborate to transmit packets through H relay sets to the sink, each relay set containing N relays. We vary H from 1 to 6, and vary N from 2 to 6. We assume a relay only accepts packets from relays in the previous set, while discard packets from relays in the same set. All relays collaborate to transmit in a round-robin manner (TDMA). We consider the following three schemes: unicast, BS-JNCC, and NB-JNCC. Both BS-JNCC and NB-JNCC use multi-path routing as described earlier, while unicast randomly selects one relay in each relay set. If a relay successfully decodes a generation of packets, it forwards a newly encoded generation of packets, otherwise it only forwards the packets that are successfully decoded. All channels in the network are assumed to be i.i.d. and obey the same fading model as described in Sec. II. Every generation contains two packets, each of $k = 800$ symbols over GF(2^4), i.e., 4 bits in each symbol. For channel coding, the non-binary irregular LDPC code we use has average column weight of 2.8 and the channel code rate is $r_c = 0.8$ [18]. For network coding, the coefficients are randomly selected from GF(2^4). Nodes in unicast only perform channel encoding and decoding. All the nodes use 16QAM modulation. The two sources generate packets with the rate of 10 packets per second (5 generations per second). We consider the following three performance metrics:

- Generation error rate (GER): the number of successfully decoded generations at the sink over the total number of generations.
- Goodput (packets per second): the number of packets in successfully decoded generations at the sink per second. In this way, we ignore the packets in partially recovered generations.

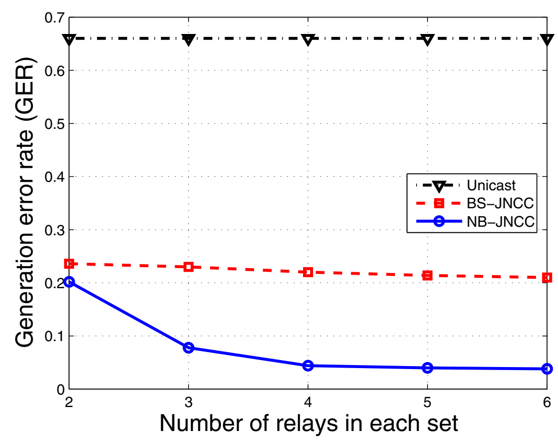


Fig. 9. Generation error rate in wireless networks with different node densities. There are three relay sets from the source to the sink, the number of relays in each set is varied from 2 to 6.

We first compare the performance of the three schemes under different channel conditions. The network contains 3 relay sets and 4 nodes in each set. Fig. 8(a) plots the GER under various schemes. We observe that NB-JNCC leads to much faster decrease in GER than BS-JNCC and the traditional unicast. Specifically, at GER of 10^{-1}, the performance gain of NB-JNCC over BS-JNCC (unicast, resp.) is about 7 dB (12 dB, resp.). Correspondingly, NB-JNCC achieves a much higher goodput than the other two schemes as shown in Fig. 8(b) because more generations can be recovered. For instance, when the SNR is 17 dB, NB-JNCC achieves a 80% (350% resp.) higher goodput than BS-JNCC (unicast, resp.).

We next compare the performance of these three schemes when varying the number of nodes in each relay set at a fixed SNR of 21 dB. This can be treated as applying NB-JNCC

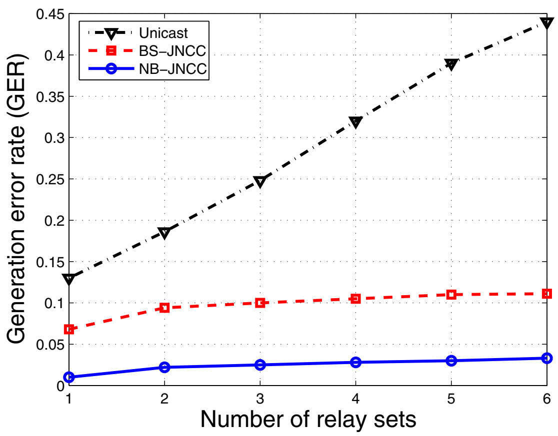


Fig. 10. Generation error rate in wireless networks with different sizes. There are one to six relay sets from the source to the sink, each relay set containing 3 nodes.

in networks with different node densities. Fig. 9 plots the GER under various schemes. We see that NB-JNCC benefits while BS-JNCC and unicast do not benefit from increasing the number relays in each set. For unicast, the lack of benefits is expected since the node density does not affect the number of hops from the source to the destination. For BS-JNCC, the lack of benefits is because the binary XOR network coding operations limit the coding diversity of the related packets and lead to duplicated packets from different paths. The ability to benefit from dense node deployment makes NB-JNCC suitable for large-scale wireless networks with high node densities. As we can see from Fig. 9, NB-JNCC with only 3 relays in each set has achieved satisfiable GER and increasing the number of relays in each set only slightly reduces the GER. This is because each relay in NB-JNCC provides innovative information, and thus only a small number relays is sufficient to obtain most of the gains.

Last, we vary the size of the network while fixing the network density. More specifically, we assume each relay set contains 3 relays and the SNR is 25 dB, while varying the number of relay sets from 1 to 6. Fig. 10 plots the GER under various schemes. It clearly shows that the GER of unicast increases linearly (and hence the goodput decreases linearly) when the number of relay sets increases, while both NB-JNCC and BS-JNCC are quite stable. This observation confirms the conclusion in [23] that network coding schemes with multi-path routing are scalable and are not sensitive to the size of the network.

In summary, NB-JNCC outperforms unicast and BS-JNCC. It achieves better error recovery and a higher goodput. Furthermore, it can take advantage of node density, and is insensitive to the size of the network. Therefore, it is suitable for large-scale wireless networks.

V. Conclusions and Future Work

In this paper, we presented a practical scheme, non-binary joint network-channel coding (NB-JNCC), for reliable communication in wireless networks. NB-JNCC seamlessly combines non-binary channel coding and random linear network coding, and uses an iterative two-tier coding scheme that we proposed to jointly exploit redundancy inside packets and across packets for error recovery. Both theory and simulation have demonstrated the significant benefits of NB-JNCC. Compared to other schemes, NB-JNCC fully exploits the spatial diversity and approaches the upper bound of outage probability with acceptable performance loss.

As future work, we plan to pursue in the following directions: 1) jointly design channel codes, network codes, modulation and routing for improved performance; 2) pursue tighter performance bounds on diversity and outage probability in large-scale networks; and 3) carry out performance comparisons with related schemes in various network topologies.

Acknowledgment

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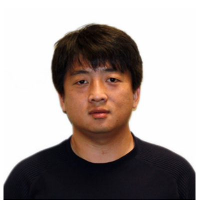

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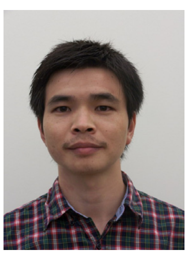
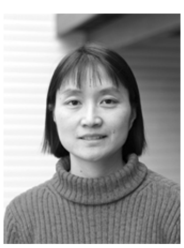


Jie Huang was born in Jiangling, Hubei, P. R. China on January 20, 1981. He received the B.S. degree in 2001 and the Ph. D. degree in 2006, from the University of Science and Technology of China (USTC), Hefei, both in electrical engineering and information science. He was a research assistant professor from July 2009 to May 2011, working with the Department of Electrical and Computer Engineering (ECE) at the University of Connecticut, Storrs. Now he is a staff DSP design engineer with Marvell Semiconductor, Santa Clara, CA.

Mr. Huang has 17 journal papers and 25 conference papers published in the area of wireless communications and digital signal processing. His recent focus is on the design and implementation of the UMTS Long-Term-Evolution (LTE) system. Mr. Huang has served as a reviewer for the IEEE TRANSACTIONS ON INFORMATION THEORY, the IEEE TRANSACTIONS ON COMMUNICATIONS, the IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS, the IEEE TRANSACTIONS ON SIGNAL PROCESSING, the IEEE JOURNAL ON SELECTED AREAS IN COMMUNICATIONS, and the IEEE COMMUNICATIONS LETTERS.

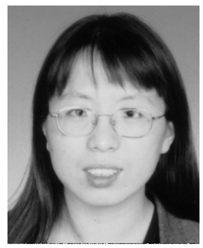


Bing Wang (M'05) is currently an Associate Professor of the Computer Science & Engineering Department at the University of Connecticut. She received her B.S. degree in Computer Science from Nanjing University of Science & Technology, China in 1994, and M.S. degree in Computer Engineering from Institute of Computing Technology, Chinese Academy of Sciences in 1997. She then received M.S. degrees in Computer Science and Applied Mathematics, and a Ph.D. in Computer Science from the University of Massachusetts, Amherst in 2000, 2004, and 2005 respectively. Her research interests are in Computer Networks, Multimedia, and Distributed Systems. She received NSF CAREER award in 2008.

Shengli Zhou (SM'11) received the B.S. degree in 1995 and the M.Sc. degree in 1998, from the University of Science and Technology of China (USTC), Hefei, both in electrical engineering and information science. He received his Ph.D. degree in electrical engineering from the University of Minnesota (UMN), Minneapolis, in 2002.

He has been an assistant professor with the Department of Electrical and Computer Engineering at the University of Connecticut (UCONN), Storrs, 2003-2009, and now is an associate professor. He holds a United Technologies Corporation (UTC) Professorship in Engineering Innovation, 2008-2011. His general research interests lie in the areas of wireless communications and signal processing. His recent focus is on underwater acoustic communications and networking.

Dr. Zhou served as an associate editor for IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS, Feb. 2005 – Jan. 2007, and IEEE TRANSACTIONS ON SIGNAL PROCESSING, Oct. 2008 – Oct. 2010. He is now an associate editor for IEEE JOURNAL OF OCEANIC ENGINEERING. He received the 2007 ONR Young Investigator award and the 2007 Presidential Early Career Award for Scientists and Engineers (PECASE).



Jun-Hong Cui (M'03) received her B.S. degree in Computer Science from Jilin University, China in 1995, her M.S. degree in Computer Engineering from Chinese Academy of Sciences in 1998, and her Ph.D. degree in Computer Science from UCLA in 2003. Currently, she is an Associate Professor in the Computer Science and Engineering Department at University of Connecticut (UConn). She also serves as the Assistant Dean for Graduate Studies and Diversity of School of Engineering at UConn.

Jun-Hong's research interests cover the design, modelling, and performance evaluation of networks and distributed systems. Currently, her research mainly focuses on algorithm, protocol and system design and development in underwater sensor networks, autonomous underwater vehicle (AUV) networks, and distributed cyber-aquatic systems. She has also been conducting active research on smart health and smart transportation systems. At UConn, she leads UbiNet (Ubiquitous Networking) Lab and UWSN (UnderWater Sensor Network) Lab. See http://www.cse.uconn.edu/~jcui/ for her recent projects and publications.

Jun-Hong is actively involved in the community as an organizer, a TPC member, and a reviewer for numerous conferences and journals. She now serves as an Associate Editor for Elsevier's *Ad Hoc Networks*. She co-founded the first ACM International Workshop on UnderWater Networks (WUWNet'06), and she is now serving as the WUWNet steering committee chair. Jun-Hong received 2007 NSF CAREER Award and 2008 ONR Young Investigator Award. She also received the United Technologies Corporation (UTC) Professorship in Engineering Innovation award at UConn in 2008 and UCLA Engineering Distinguished Young Alumnus Award in 2010.

Peter Willett (F'03) received his BASc (Engineering Science) from the University of Toronto in 1982, and his PhD degree from Princeton University in 1986. He has been a faculty member at the University of Connecticut ever since, and since 1998 has been a Professor. His primary areas of research have been statistical signal processing, detection, machine learning, data fusion and tracking. He has interests in and has published in the areas of change/abnormality detection, optical pattern recognition, communications and industrial/security condition monitoring.

Dr. Willett is editor-in-chief for IEEE TRANSACTIONS ON AEROSPACE AND ELECTRONIC SYSTEMS, and until recently was associate editor for three active journals: IEEE TRANSACTIONS ON AEROSPACE AND ELECTRONIC SYSTEMS (for Data Fusion and Target Tracking) and IEEE TRANSACTIONS ON SYSTEMS, MAN, AND CYBERNETICS, parts A and B. He is also associate editor for the *IEEE AES Magazine*, editor of the AES Magazines periodic Tutorial issues, associate editor for ISIF's electronic *Journal of Advances in Information Fusion*, and is a member of the editorial board of *IEEE Signal Processing Magazine*. He has been a member of the IEEE AESS Board of Governors since 2003. He was General Co-Chair (with Stefano Coraluppi) for the 2006 ISIF/IEEE Fusion Conference in Florence, Italy, Program Co-Chair (with Eugene Santos) for the 2003 IEEE Conference on Systems, Man, and Cybernetics in Washington DC, and Program Co-Chair (with Pramod Varshney) for the 1999 Fusion Conference in Sunnyvale.